

Variational collision integrators

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Abstract

We use variational techniques to analyze the problem of rigid-body dynamics with impacts. The theory of smooth Lagrangian mechanics is extended to a nonsmooth context appropriate for collisions and it is shown in what sense the system is symplectic and satisfies a Noether-style momentum conservation theorem.

Discretizations of this nonsmooth mechanics are developed by using the methodology of variational discrete mechanics. This leads to variational integrators which are symplectic-momentum preserving and are consistent with the jump conditions given in the continuous theory. Specific examples of these methods are tested numerically, and the long-time stable energy behavior typical of variational methods is demonstrated.