Analysis of a Few Numerical Integration Methods for the Langevin Equation

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Abstract

We study the position recurrence relation of several existing numerical integrators for the Langevin equation and use the modified equation approach to analyze their accuracy. We show that, for the harmonic oscillator, the BBK integrator converges weakly with order 1 while the vGB82 and Langevin Impulse (LI) integrator converge weakly with order 2. We also study a restricted class of velocity definitions — those that lead to explicit starting procedures. We show that some recurrence relations exact for constant force, can achieve the exact virial relation by a proper definition of velocity, extending the result of Pastor et. al on the analysis of BBK integrators in 1988.