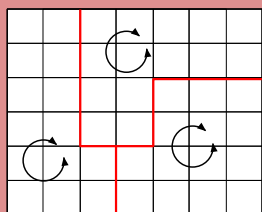
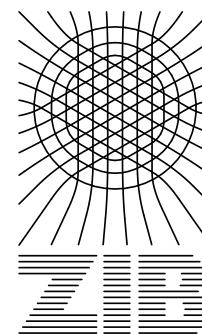


# Characterization of Transition States via Metastable Fuzzy Sets

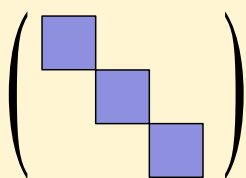
M.Weber, T.Galliat



$\Omega$

**Problem:** After discretization of  $\Omega$  into  $m$  boxes find a decomposition into almost invariant sets based on a stochastic operator. Identify transition boxes.

**Idea:** For every box compute the grade of membership to each of the  $k$  almost invariant sets, i.e. use metastable fuzzy sets instead of metastable crisp sets.



$(m,m)$ -transition matrix  $T$   
"hidden" block diagonally dominant

spectrum of  $T$  with a gap after  $k=3$   
1.0 0.99 0.98 | 0.69 0.68 ...

$$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \begin{pmatrix} * \\ \vdots \\ * \end{pmatrix} \begin{pmatrix} * \\ \vdots \\ * \end{pmatrix} \text{ basis of } I_T$$

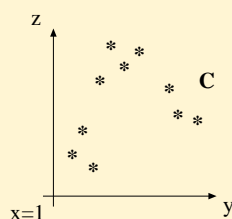
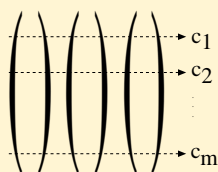
$$\chi \in I_T \longrightarrow T\chi \approx \chi$$

**Properties of a solution  $S = \{\chi_1, \dots, \chi_k\}$**

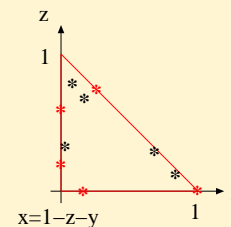
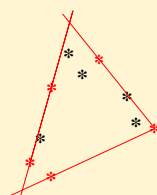
- a)  $\chi_i \in [0, 1]^m \cap I_T, \quad i = 1, \dots, k$
- b)  $S$  is linear independent
- c)  $\sum_{i=1}^k \chi_i = \mathbf{1}_\Omega$
- d)  $\chi_A, \chi_B \in [0, 1]^m \cap I_T, \quad \chi_A + \chi_B = \chi_i$   
 $\Rightarrow \{\chi_A, \chi_B\}$  linear dependent

## Linear transformation of a basis of $I_T$ into a feasible solution $S$

1. Projection of a basis of  $I_T$

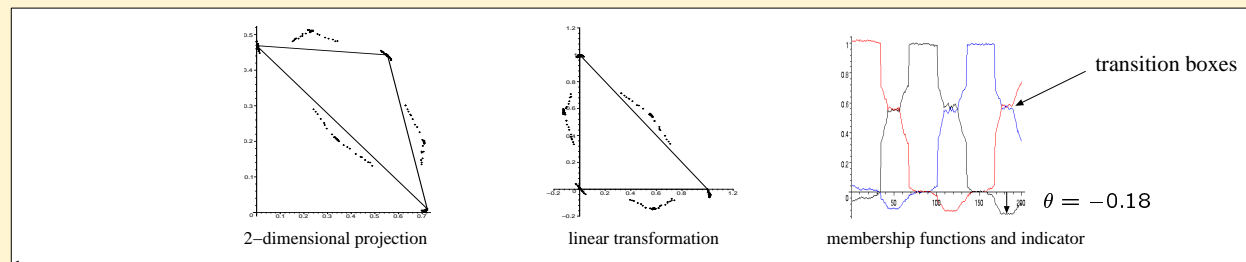


2. Find  $(k-1)$ -simplex  $V$  covering  $C$  (see prop. a-c), each facet of  $V$  includes at least  $(k-1)$  different points of  $C$  (see prop. d). Transform  $V$  into an unit simplex yielding a projection of  $S$ .



## Indicator for the uniqueness of $S$

The choice of an outer simplex for the linear transformation is not unique if the convex hull of  $C$  is not a simplex. Therefore choose a maximal **inner** simplex and use the lowest value of  $\chi_i$  as an indicator for the uniqueness of  $S$  (example:  $m=200, k=3$ ).



## References

- [1] P. Deuffhard, W. Huisinga, A. Fischer and Ch. Schlitte: Identification of almost invariant aggregates in nearly uncoupled Markov chains. *Linear Algebra and its Applications*, 315: 39–59, 2000.
- [2] T. Galliat, P. Deuffhard, R. Roitzsch and F. Cordes: Automatic Identification of Metastable Conformations via Self-Organized Neural Networks. *Computational Methods for Macromolecules: Challenges and Applications, Proceedings of 3rd International Workshop on Algorithms for Macromolecular Modelling, New York, Oct. 12–14, 2000*, T. Schlick and H.H. Gan, eds., *Lecture Notes in Computational Science and Engineering*, Vol.24, Springer, 2002.
- [3] M.Weber, T. Galliat: Characterization of Transition States in Conformational Dynamics using Fuzzy Sets. *ZIB-Report 02–12, March 2002*.

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